

MATHEMATICAL TECHNIQUES

Duration: 2 hours

Max. Marks: 80

INSTRUCTIONS:

1. All questions are compulsory.
2. Start each new question on fresh page.
3. Figures to the right indicate full marks
4. Use of calculators is allowed.

Q.1 Attempt the following: (5 X 4 = 20)

- a. If A (7,5) and B(x,0), find the possible values of x so that $I(AB)=13$.
- b. Find the derivative of the following w.r. t x if
 - i) $y = \frac{x+1}{x}$
 - ii) $y = (4x^3-3)(3x+5)$
- c. If $f(x) = (x+2)(5-x)(x-1)$, $-2 \leq x \leq 5$ and $f(-1), f(2), f(5)$.
Also find x if $f(x) = 0$
- d. Evaluate the following integrals:

- i) $\int \frac{x^2-5x+6}{x-2} dx$
- ii) $\int (x+3)(x-2) dx$

OR

Q.I Attempt the following: (5 X 4 = 20)

- w. The point P (7, a) lies on the line joining A (-5,2) and B(3,6). Find the ratio in which P divides AB and hence find a.
- x. Find the derivative of the following w. r. t x if
 - i) $y = (x^2 + 2)(x - 5)$
 - ii) $y = x^3 e^x - x^{-6}$
- y. If $f(x) = 3x - 1$ and $g(x) = x^2 + 1$ Find $f(g(x))$ and $g(f(x))$.
- z. Evaluate the following integrals:

- i) $\int_2^3 (e^x - 2x) dx$
- ii) $\int_0^6 (x+1)(x-2) dx$

Q.2 Attempt the following: (5 X 4 = 20)

- a. Find (i) $\lim_{x \rightarrow 2} \frac{x^2+x+1}{x^2-x+3}$ (ii) $\lim_{x \rightarrow 3} \frac{x^2-9}{x-3}$
- b. The cost of manufacturing x toys is given by $C = x^2 + 5x + 5$, where x denotes the number of toys manufactured. Find the total cost, average cost and marginal cost when 10 toys are made.
- c. Solve graphically, the following L.P. problem.
Minimize $z = 6x + 7y$
subject to : $2x + 3y \geq 12$, $2x + y \geq 8$, $x \geq 0, y \geq 0$
- d. The Utility functions for a group of consumers is given by:
 $U(x_1, x_2) = 5x_1^2 + 3x_1x_2 + x_2^3 + 500$.
Find the marginal utilities at $x_1 = 10$, and $x_2 = 2$

OR

Q.II Attempt the following: (5 X 4 = 20)

- w. Examine the continuity of at f at $x = 3$ if

$$f(x) = \begin{cases} x^2 + 1 & 0 \leq x \leq 3 \\ 3x + 1 & 3 < x \leq 5 \end{cases}$$
- x. Find out when the following functions are (i) increasing (ii) decreasing.
 $f(x) = x^3 - 9x^2 + 15x + 20$

- z. Find the partial derivatives of $f(x, y) = x^2 + 5xy + y^2 - 3x + 10$, at the point (1,4).

- Q.3 Attempt the following: (5 X 4 = 20)
- If the compound rate of interest is 16% p.a. payable half yearly, find effective rate
 - Show that (3,-5), (4, 3) and (11,-4) are the vertices of an isosceles triangle.
 - If the Marginal Revenue function for a certain product is $MR = 12 - 3x^2 + 4x$. Find the Revenue function and the corresponding demand function.
 - If $y = e^x + \frac{1}{x} - 3x$ Find $\frac{d^2y}{dx^2}$.

OR

- Q.III Attempt the following: (5 X 4 = 20)
- In how many years, the amount of money will be double the principal at simple interest of 12% per annum?
 - If A(-3,1) and B(4,7) and Q divides the segment AB externally in the ratio 3:2, find the coordinates of Q.
 - The demand function for a certain commodity is $p = 100 - 5x$. Find the consumer's surplus at $x = 4$
 - A firm produces an output of x tons at a total cost $C = x^3 - 4x^2 + 7x$. Find the output at which the average cost is minimum.

- Q.4 Attempt the following: (5 X 4 = 20)
- Find the present value of an annuity of Rs. 2000, paid at the end of each year for 4 years, at 11% compounded annually.
 - Find the equation of the line passing through the point (3,2) and parallel to the line $5x+4y=500$.
 - If the demand function is given by $D = 25 - 3p - p^2$, Find price elasticity of demand when $p=2$.
 - Evaluate: $\int \left[\frac{x^2+x-12}{x-3} \right] dx$

OR

- Q.IV Attempt the following: (5 X 4 = 20)
- A bank offers fixed deposits for 5 years under the following schemes:
 - At 15% if the interest to be calculated half yearly.
 - At 12% if the interest to be calculated quarterly.State which scheme is more beneficial to the public?
 - Obtain the equation of a straight line having slope $-1/2$ and passing through the point (2,6).
 - The cost function for x units of a product produced and sold by a company is $C(x) = 250 + 0.005x^2$ and the revenue is given as $R(x) = 4x$. Find how many items should be produced to maximize the profit. What is the maximum profit?.
 - Evaluate: $\int_0^2 (3x^2 - x^3) dx$