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FYBCOM, Semester II, Semester End Examination – April 2016

MATHEMATICAL TECHNIQUES

Duration: 2 hours

Max.Marks : 80

Instructions: 1) All question are compulsory (choice is internal)

2) Start each new question on a fresh page

3) Figures to the right indicate full marks

4) Use of calculators not allowed

Q.1 Attempt the following:

(5 x 4=20)

a) Prove that the points (4,3), (7,-1) and (9,3) are the vertices of an isosceles triangle.

b) Find the derivative of the following w.r.t. x if

i) $y = \frac{3 \log x + 1}{x^2}$ and ii) $y = \log(3x^2 + 5x - 1) + \sqrt{x^2 - 1}$

c) Find x if $f(x + 1) = f(x + 2)$ for $f(x) = 1 + x - x^2$

d) Evaluate the following integrals:

i. $\int x(x + 3)(x - 5) dx$

ii. $\int (5x^4 + 6e^x + 7^x + 5) dx$

OR

Q.I Attempt the following:

(5 x 4=20)

w) If A(2,-4) and B(-1,-7) are 2 points, find the equation of the line passing through origin and parallel to AB.

x) Find $\frac{dy}{dx}$ if i) $y = 7^x(x^4 - x^3 + 5x^2 + 2)$

ii) $y = \frac{x^2 - x + 5}{x - 3}$

y) Find $f(g(x))$ and $g(f(x))$ if $f(x) = 3x - 1$ and $g(x) = x^2 + 1$

z) Evaluate the following integrals:

$$\int_1^3 \frac{x^2 + 4x + 3}{x + 1} dx$$

Q.2 Attempt the following:

(5 x 4=20)

a) Evaluate the following if they exist

i) $\lim_{x \rightarrow 2} \left[\frac{1}{x^2 + x - 6} + \frac{1}{x^2 - 9x + 14} \right]$ ii) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - x - 2}$

b) The demand function p in terms of quantity demanded (D) is given by $p = 30 + 12D - 4D^2$. Find total revenue, average revenue and marginal revenue when the demand is 4 units.

- c) Food I contains 6 units of vitamin A per gram, 7 units of vitamin B per gram and cost Rs.12 per gram. Food II contains 8 units of vitamin A per gram 12 units of vitamin B per gram and cost Rs.20 per gram. The daily minimum requirement of vitamin and vitamin B are 400 units and 480 units respectively. Find the minimum cost of the mix of these foods.
- d) Find the partial derivatives of the functions $f(x, y) = x^2 + 2xy + y^2$ at the point (1, 2)

OR

Q.II Attempt the following:

(5 x 4=20)

- w) Discuss the continuity of the following function at $x = 4$

$$f(x) = \begin{cases} x^2 + x + 2 & ; 2 \leq x \leq 4 \\ 3x + 4 & ; 4 < x \leq 6 \end{cases}$$

- x) Find the values of x for which the function $f(x) = x^3 - 75x + 10$ is increasing
- y) Solve graphically

$$\text{Max } z = 2x + 5y$$

$$\text{s.t. } 2x + 4y \leq 8$$

$$3x + y \leq 6$$

$$x + y \leq 6$$

$$\text{And } x \geq 0, y \geq 0$$

- z) For the cost function $C(x, y) = 3x^2 + 2xy + y^2 + 10$, for two commodities x and y, find the marginal cost at $x = 1$ and $y = 5$.

Q.3 Attempt the following:

(5 x 4=20)

- a) Find the amount if Rs.5000 is invested at 12% simple interest for 9 months.
- b) In what ratio does the y-axis divide the join of (3, 5) and (6, 7)? Is the division internal or external? Find the coordinates of the point of division.
- c) The marginal demand $MD = 2 - 3p^2$ and $D = 79$ when $p = 3$. Find the demand function when $p = 2$.
- d) Find $\frac{dy}{dx}$ if i) $y = 7x^5 - 4x^2 + 6$

$$\text{ii) } y = 2x^3 + 5^x$$

OR

Q.III Attempt the following:

(5 x 4=20)

- w) Find the compound interest and amount after 3 years on a principal of Rs.15,000 at 10% p.a
- x) If A(1,-2) and B(4,y) find the possible values of y such that $l(AB) = 5$
- y) The marginal revenue of selling x items is $MR = x^2 + x - 1$. Find the total revenue and average revenue at $x = 6$.

z) Find $\frac{dy}{dx}$ if $y = \frac{1}{\sqrt{x^2+5} + \sqrt{x^2+1}}$.

Q.4 Attempt the following:

(5 x 4=20)

- a) Find the amount for the ordinary annuity with periodic payments as Rs.100 at the rate of interest 12% p.a. for 1 year if period of payment is yearly.
- b) The point P(7,a) lies on the line joining A(-5,2) and B(3,6) find the ratio and hence find a .
- c) Find the elasticity of y w.r.t. x if $y = 1 + 2x - x^2$ for i) $x = 1$ and ii) $x = 10$
- d) Evaluate the following integrals:
- i. $\int 10^{(7x+8)} + (2x - 1)^5 dx$
- ii. $\int (2x + 3)^2 dx$

OR

Q.IV Attempt the following:

(5 x 4=20)

- w) A bank decided to collect fixed deposit at the rate of 10 % to be compounded quarterly basis. Find the effective rate of interest.
- x) By using slopes show that (2,1) (6,5),(4,7) are the vertices of a right angled triangle.
- y) Examine the following function for maximum and minimum $P = x^4 - 4x$.
- z) Evaluate the following integrals:

i. $\int \frac{x^2+4x-5}{x(x+5)} dx$

ii. $\int e^{2x+7} + \frac{1}{5x-9} dx$