

MATHEMATICAL TECHNIQUES

Duration: 2 hours

Max.Marks : 80

Instructions: 1) All question are compulsory (choice is internal)

2) Start each new question on a fresh page

3) Figures to the right indicate full marks

4) Use of calculators not allowed

Q.1 Attempt the following:

(5 x 4=20)

a) Test the validity of the following statements:

“If he studies engineering he will get a good job. If he gets a good job, he will get a good wife. He did not get a good wife. Therefore he did not study engineering.”

b) If $A = \{x|x^2 - 11x + 18 = 0\}$, $B = \{x|(x - 1)(x - 2)(x - 7) = 0\}$, $C = \{x|x^2 - 9x + 14 = 0\}$
 Verify that i) $A \cap (B - C) = (A \cap B) - (A \cap C)$

ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

c) Find x if $\begin{vmatrix} 5 & -3 & 7 \\ 2 & 1 & 2 \\ 9 & -1 & x \end{vmatrix} = 0$

d) The sum of three consecutive numbers in A.P. is 24 and their product is 440, find the numbers.

OR

Q.I Attempt the following:

(5 x 4=20)

w) Construct the truth table for $(p \vee \sim q) \rightarrow (\sim p \wedge q)$

x) In a class of 200 students, 120 play soccer and 100 play cricket. If each student have to play atleast one game find the no. of students i) who play both games and ii) who play cricket only.

y) Without actual expansion as far as possible prove that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$$

z) Find the sum of n terms of $0.5 + 0.55 + 0.555 + \dots - n \text{ terms.}$

Q.2 Attempt the following:

(5 x 4=20)

a) Find the number of different words containing all the letters of the word “VILHENA” such that (1) All the vowels are together (2) begin and end with consonant.

b) A father-son duo wishes to save Rs.10000 each in tax saving bonds of two types, TSI and TSII. The details of their savings are listed below. Find the value of each type of bonds.

	Tax saving bonds	
	TS I	TS II
Father	5	10
Son	7	6

c) What number must be added to each term of the ratio 5:37 to make it equal to 1:3 .?

d) Three numbers whose sum is 15 are in A.P. If 12, 10 and 8 is added to them respectively the results are in G.P. Find the numbers.

OR

Q.II Attempt the following:

(5 x 4=20)

w) Find the no. of words which can be formed with the letters of each of the words

i) BALLOON

ii) INDIFFERENCE

x) Find the adjoint of the matrix $\begin{bmatrix} 3 & 3 & 4 \\ 2 & -3 & 5 \\ 0 & -1 & 1 \end{bmatrix}$

y) The ratio of two numbers is 5:6. On adding 3 to each of these numbers, the ratio becomes 6:7. Find the numbers.

- z) In a G.P. the fourth and seventh terms are 24 and 81 respectively. Find first and common ratio.

Q.3 Attempt the following:

(5 x 4=20)

- a) If $X = \{A, B, C, D, E, G, H, I, L, M, N, O, R, S, T, Y, Z\}$
 $A =$ set of letters of the word "LOGARITHM"
 $B =$ set of letters of the word "THEORY"

$C =$ set of letters of the word "THEOREMS"

Verify that i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ii) $B - C = (B' \cup C)'$

- b) From 6 lawyers and 4 doctors, a committee of 5 is to be formed. In how many ways can this be done when the committee contains at least 3 doctors?
 c) If 30 men working 8 hours a day can complete a piece of work in 24 days. In how many days 18 men working 10 hours a day will complete the double work.
 d) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ find the matrix X such that $2X + 4A - 3B = 0$

OR

Q.III Attempt the following:

(5 x 4=20)

- w) In a class of 50 students, 30 students take Economics, 25 students take Mathematics and 10 took both. Find the no. of students who took neither of the two subjects.
 x) In a group of 20 students, 13 students offered Mathematics and 7 have offered Statistics. In how many ways can a committee of 5 students be formed so that it will have a majority of students offering Mathematics?
 y) The ratio of students in Arts, social science and commerce faculties are in the ratio of 7:5:2. If the number of students in social sciences be 560. Find the number of students in Arts and Commerce faculties?

- z) Find the inverse of the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 4 & 4 \end{bmatrix}$.

Q.4 Attempt the following:

(5 x 4=20)

- a) Akhil saves 20 % of his earnings. Due to increase in cost of living by 15% he could now save only Rs.780. Find the monthly earnings.
 b) Find whether following statement is tautology or contradiction:
 $(p \rightarrow q) \rightarrow [(q \rightarrow r) \rightarrow (p \rightarrow r)]$
 c) Find the value of ${}^8C_5 + {}^7C_4 + {}^6C_3 + {}^6C_2$.
 d) A money lender lends Rs.1000 and charges an overall interest Rs.140 for 12 months. He recovers the loan and interest by 12 monthly instalments each less by Rs. 10 than the preceding one. Find the amount of first instalment.

OR

Q.IV Attempt the following:

(5 x 4=20)

- w) The list price of an article is Rs.120. If a discount of 6% is allowed. Find the amount payable by the customer.
 x) Prove that the following pairs of statements are equivalent:
 $\sim r \rightarrow \sim(p \wedge q), \sim(q \rightarrow r) \rightarrow \sim p$
 y) What is the value of n when ${}^nP_2 = 30$?
 z) A man wants to repay Rs.6050 in five monthly instalments such that each instalment is three times the value of the previous instalment. Find the value of the first instalment