

DISCRETE MATHEMATICS**Duration: 2 Hours****Total Marks: 50****Instructions:**

1. Figures to the right indicate maximum marks
2. All questions are compulsory

Q.1 Answer the following**a) Answer the following questions****(1X5=5)**

- i. Write converse, inverse and contra-positive statement of the implication
if $x = 4$ then $x^2 - 16 = 0$
- ii. If $f(x) = x^2$, find $f(f(x))$
- iii. Draw logic gate diagram for $\bar{x}_1 + x_2$
- iv. If $A = \{x \in \mathbb{N} \mid 1 < x < 9\}$ and $B = \{x \in \mathbb{N} \mid x \text{ is even and } x < 10\}$
Write $A - B$ and $B - A$
- v. ${}^6C_4 - {}^5C_4 = \underline{\hspace{1cm}} ? \underline{\hspace{1cm}}$

b) Solve the following**(1X5=5)**

- i. Convert $(10)_2$ into Decimal number
- ii. Convert 0.3125 into Binary number
- iii. Convert 56 into Hexadecimal number
- iv. Convert 16 into octal number
- v. Convert $(AB)_{16}$ into decimal number

Q.2 Answer the following (Any two)**(5X2=10)**

- a) Use principle of mathematical induction to prove that

$$\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \cdots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}, \quad n \geq 2, n \in \mathbb{N}$$

- b) Use Binomial theorem to prove that

$${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n \text{ and}$$

$${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots + (-1)^n {}^nC_n = 0$$

- c) Prove that $(p \rightarrow q)$ and $[(\sim p \vee q) \wedge (\sim q \vee p)]$ are not equivalent

- a) Prove the following by using properties of Boolean algebra
 $(x + y) \cdot (x + z) \cdot \overline{(\bar{x} \cdot y)} = x$
- b) Use basic logic gates to draw logic diagram for the following
 NAND, NOR, XOR, $(x_1 + \bar{x}_2) * \bar{x}_1$, $(\bar{x}_1 + \bar{x}_2) * (\bar{x}_2)$
- c) Write the definitions of the following in set builder form
- Union of two sets
 - Intersection of two sets
 - Complement of a set
 - Difference of two sets
 - Cartesian product

Q.4 Answer the following (Any two)

(5X2=10)

- a) If $U = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$, U is a universal set and $A = \{2,3,5,7,11,13\}$ and $B = \{2,4,6,8,10,12,14\}$, then prove that
- $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$
 - $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$
 - $A - B = A \cap \bar{B}$
- b) There are 6 boys and 6 girls in a committee. A group of 4 should be sent abroad for a presentation. In how many ways can this be done if
- There should be equal number of boys and girls
 - The group consists of atmost 4 girls
 - Exactly 4 boys
 - Atleast 3 girls should be there in the group
 - No restriction on the number of boys and girls
- c) Justify why the following relations defined on $A = \{1,2,3,4,5,6\}$ are not Equivalence relations
- $R_1 = \{(1,1), (2,2), (4,4), (5,5), (6,6)\}$
 - $R_2 = \{(a,b) | a \leq b\}$
 - $R_3 = \{(a,b) | a + b = 2\}$
 - $R_4 = \{(a,b) | a^2 = b\}$
 - $R_5 = \{(1,2), (2,3), (1,3)\}$

Q.5 Answer the following (Any two)

(5X2=10)

- a) Draw the graph of Greatest Integer Function. Consider the functions $f, g: \mathbb{N} \longrightarrow \mathbb{N}$ defined by $f(x) = 2x$ and $g(x) = 2x - 1$ prove that both f and g are one-one, but both are not onto
- b) Define equivalence relation. Prove that the relation $R = \{(a,b) | a - b \text{ is divisible by } 7\}$ defined on \mathbb{N} is an Equivalence relation
- c) Find the languages $L(G)$ generated by the grammar with $V = \{S, A, B\}$, $P = \{x, y\}$ and $P = \{S \longrightarrow xB, B \rightarrow y, B \rightarrow yA, A \rightarrow xB\}$

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