

DISCRETE MATHEMATICS

**Duration: 2 Hours**

**Total Marks: 50**

**Instructions:**

1. Figures to the right indicate maximum marks
2. All questions are compulsory

**Q.1 Answer the following**

**a) Attempt the following questions**

**(1X5=5)**

- i. Write the truth table for Implication and Disjunction.
- ii. Let  $f: R \rightarrow R$  be defined by  $f(x) = 2x$ , prove that  $f$  is one-one
- iii. Write the logic symbols for XOR and NAND gates
- iv. If  $A = \{x \in N \mid x^2 - 6x - 40\}$  and  $B = \{x \in N \mid x^2 - 7x + 10 = 0\}$   
Write  $A \times B$  and  $B \times A$  (i.e. find the Cartesian product)
- v. Find the number of ways in which a four digit number can be formed by using the digits 0,1,2,3,4,5,6,7,8,9 if repetition of digits is allowed

**b) Solve the following**

**(1X5=5)**

- i. Convert 227 in to binary form
- ii. Convert  $(DEF)_{16}$  in to decimal form
- iii. Convert 22.625 into binary form
- iv. Convert 28 into octal form
- v. Convert 112 into hexadecimal form

**Q.2 Answer the following (Any two)**

**(5X2=10)**

- a) Use principle of mathematical induction to prove that

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left( \frac{n(n+1)}{2} \right)^2, \quad n \in N$$

- b) For the binomial expression  $(2x + 4y)^7$ , find its middle terms, find 6<sup>th</sup> term, find the coefficients of  $x^2y^5, x^4y^3$  and write the binomial expansion for the same
- c) Prove that  $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$  is a tautology

**Q.3 Answer the following (Any two)**

**(5X2=10)**

- a) Prove the following using properties of Boolean algebra

$$(a + b) \cdot (\bar{b} + c) + b \cdot (\bar{a} + \bar{c}) = a \cdot \bar{b} + a \cdot c + b$$

- b) Write truth tables XOR, NOR, NAND, NOT and AND gate
- c) A survey was conducted in a city to study the preference of three brands of soaps, Liril, Lux and Rexona. The total number of persons interviewed was 200. 92 people

said that they use Liril soap, 86 use Lux and 83 use Rexona. Also 25 of them use Liril and Lux, 27 use Liril and Rexona soap, 26 use Lux and Rexona and 9 use all the three. Find the number of people using

- i. Liril only
- ii. Lux only
- iii. Rexona only
- iv. Liril or Lux
- v. Rexona and Lux

Q.4 Answer the following (Any two)

(5X2=10)

- a) Write the following in Set builder form
  - i.  $A = \{3, 9, 27, 81, 243\}$
  - ii.  $B = \{2, 4, 6, 8, 10, 12, 14\}$
  - iii.  $C = \{2, 3, 5, 7, 11, 13\}$
  - iv.  $D = \left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7} \right\}$
  - v.  $E = \{\pm 1, \pm 3, \pm 5, \pm 7, \pm 9\}$
- b) Prove that  ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$  for  $r \leq n$
- c) Prove that the relation  $R = \{(a, b) | a \geq b\}$  defined on set of all real numbers  $R$  is a partially ordered relation

Q.5 Answer the following (Any two)

(5X2=10)

- a) Consider the functions  $f: R \rightarrow R$  and  $g: R \rightarrow R$  defined by  $f(x) = \frac{1}{1+x}$ ;  $x \neq -1$  and  $g(x) = \frac{1+x}{x}$ ;  $x \neq 0$  then
  - i. Find  $f \circ f$
  - ii. Find  $g \circ g$
  - iii. Find  $f \circ g$
  - iv. Prove that  $f$  is one-one
  - v. Prove that  $g$  is one-one
- b) Write the following relations defined on  $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$  in roster form
  - i.  $R_1 = \{(a, b) | a = b\}$
  - ii.  $R_2 = \{(a, b) | a = b^2\}$
  - iii.  $R_3 = \{(a, b) | b = a^2\}$
  - iv.  $R_4 = \{(a, b) | a + b = 4\}$
  - v.  $R_5 = \{(a, b) | a^2 + b^2 = 1\}$
- c) Define the following
  - i. Language
  - ii. Length of a string
  - iii. Empty string
  - iv. Regular languages
  - v. Regular expression

XXXXXXXXXXXXXXXXXXXXXXXXXX