

## MATHEMATICAL TECHNIQUES

Duration: 2 hours

Max. Marks: 80

## INSTRUCTIONS:

1. All questions are compulsory.
2. Start each new question on fresh page.
3. Figures to the right indicate full marks
4. Graph paper and Log tables are provided with request.

Q.1 Attempt the following: (5 X 4 = 20)

a If P (2, -3) and Q (5, y), find the value of y so that l(PQ) = 5.

b Find the derivative of the following w.r.t x if

i)  $y = x^2 - 7x + e^x$

ii)  $y = (x - 5) \log x$

c If  $f(x) = x + 1$  and  $g(x) = x$ , find  $f(g(x))$  and  $g(f(x))$ .

d Evaluate the following integrals:

i)  $\int (20x^7 + 5x + 1) dx$

ii)  $\int (x + 3)^5 dx$

OR

Q.1 Attempt the following: (5 X 4 = 20)

w Find the coordinates of the point dividing the segment joining the points (1, 2), (3, 4) internally in the ratio 2:5.

x Find the derivative of the following w.r.t x if

i)  $y = x^5 + x^{-3} - \sqrt{x}$

ii)  $y = \frac{e^x}{x}$

y If  $f(x) = x^2 + 3$  and if  $f(x + 1) = f(x - 1)$ , find x.

z Evaluate the following integrals:

i)  $\int_1^3 (12x + 5) dx$

ii)  $\int_0^6 \left( e^x + \frac{1}{x} \right) dx$

Q.2 Attempt the following: (5 X 4 = 20)

a Find  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - x - 2}$ .b If the total cost function is given by  $C = 4x^2 + 5x + 2$ , find the average cost and marginal cost when  $x = 2$ .

c Solve graphically, the following L.P. problem.

Minimize  $z = 30x + 10y$

subject to:  $x + 2y \leq 8, x + 2y \geq 4, 6x + 4y \geq 12, x \geq 0, y \geq 0$ .

d For the function  $f(x, y) = x^2 + 3xy + y^2$ . Find  $f_x$  and  $f_y$  at (1, 2).

OR

Q.II Attempt the following: (5 X 4 = 20)

w If f is continuous at  $x = 2$  where

$$f(x) = x^2 - x + 1 \quad 0 \leq x \leq 2$$

$$= 5x + a \quad 2 < x \leq 4$$

find a.

x Find out when  $f(x) = x^2 - 4x + 7$  is increasing and decreasing on R.

y Solve graphically the following L.P. P  
 Maximize  $z = 9x + 13y$   
 subject to :  $2x + 3y \leq 18$ ,  $2x + y \leq 10$ ,  $x \geq 0$ ,  $y \geq 0$ .

z For  $f(x, y) = x^2 + 2xy - y^2 + 10$ , find  $f(x, y)$ , when  
 (i)  $x = 2$  (ii)  $y = 1$  (iii)  $(x, y) = (2, 1)$ .

Q.3 Attempt the following:

(5 X 4 = 20)

a In how many years, the amount of money will be double the principal at simple interest of 12% per annum?

b Find the equation of line passing through the points A (1, 6) and B (-5, 0).

c If the Marginal Cost function for a certain product is

$MC = 3x^2 + 4x + 5$ . Find the Cost function, if the fixed cost is 150.

d Find the extreme values of the function  $f(x) = x^4 - 4x$ .  
 OR

Q.III Attempt the following:

(5 X 4 = 20)

w Find the present value of an annuity of Rs. 3,500 per year for 3 years at 12% per annum.

x Find the equation of the line whose x-intercept is 4 and which is perpendicular to the line  $x - 2y + 4 = 0$ .

y The demand function for a certain commodity is  $p = 100 - 5x$ . Find the consumer's surplus at  $x = 4$ .

z If  $y = \log x + 3x^2 + 100$  Find  $\frac{d^2y}{dx^2}$ .

Q.4 Attempt the following:

(5 X 4 = 20)

a A bank has decided to collect Fixed Deposit at the rate of 10% p.a, to be compounded half yearly basis. Find the effective rate of interest.

b Show that (3, -5), (4, 3) and (11, -4) are the vertices of an isosceles triangle.

c If marginal revenue (MR) = 20 and elasticity of demand w.r.t price is 2, find the price.

d Evaluate:  $\int (x^2 - 3x + e^x + 3^x) dx$

OR

Q.IV Attempt the following

(5 X 4 = 20)

w Find the sum borrowed by Mohit from a bank on compound interest at 5% per year, to be calculated annually, if he had to pay back Rs. 26,460 after 2 years

x Obtain the equation of a straight line having slope  $-1/4$  and passing through the point (-2, 4).

y A manufacturer sell  $x$  items at a price  $p = 210 - x$ . The total cost of producing these items is  $C(x) = x^2 + 10x + 15$ . Find  $x$  for which the profit is maximum.

z Evaluate:  $\int_0^2 (1 + x^2 - x^3) dx$