Vidya Vikas Mandals

Shree Damodar college of commerce and Economics, Margao, Goa

FYB.COM, SEM II, SUPPLEMENTARY EXAMINATION, MAY/JUNE 2015

MATHEMATICAL TECHNIQUES

Duration:2hours **Max.Marks: 80**

Instructions: 1) All questions are compulsory (choice is internal)
2) Start each new question on a fresh page
3) Figures to the right indicate full marks
4) Use of calculators not allowed
5) Graph paper and logbook provided on request.

Q.1 Attempt the following: **(5 x 4=20)**

a) Show that the points A (5, 4), B(2, 3) and c(1, 0) are the vertices of a isosceles triangle.

b) Find $\frac{dy}{dx}$ if i) $y = \frac{5e^x-1}{3^x-2}$

ii) If $y = (5^x + x^2 + 1)^3$

c) If $f(x) = x^2 + 5x + 7$, solve the equation $f(x) = f(x + 1)$.

d) Evaluate $(i) \int_1^3 \frac{x^2+4x-5}{x(x+5)}\, dx$

(ii) $\int_0^6 (x + 3)(x - 2)\, dx$

OR

Q.I Attempt the following: **(5 x 4=20)**

w) If the distance between the points (a,-5) and (2,a) is 13, find a.

x) Find $\frac{dy}{dx}$ if $i) y = \frac{x^3-2e^x}{4x^3+8^x}$

$ii)\ y = 7^x(x^4 - x^3 + 5x^2 + 2).$

y) If $f(x) = \frac{x-4}{4x-1}$, show that $f(f(x)) = x$.

z) Evaluate i) $\int \frac{1}{5x-9}\, dx$

ii) $\int x(2x - 1)^2\, dx$

Q.2 Attempt the following: **(5 x 4=20)**

a) Evaluate the following if they exist

i)$\lim_{x\to 3} \frac{x^2-4x+3}{2x^2-3x-9}$ ii) $\lim_{x\to 2} \left[\frac{1}{x-2} - \frac{1}{x^2-3x+2}\right]$

b) The total cost function is given by $c = x^2 + x + 20$. Find the average cost, marginal cost and marginal average cost when $x = 10$.

c) Find the coordinates of the point which divides the segment AB internally in the ratio 4:3 if A≡ (-3,4) and B≡ (5,2).

d) Find the partial derivatives of the functions $f(x; y) = \frac{xy}{x^2+y^2}$ at the point (1, 2).

OR

Q.II Attempt the following: **(5 x 4=20)**

w) Discuss the continuity of the following function at $x = 0$

$$f(x) = \begin{cases} \frac{\sqrt{4+x} - \sqrt{4-x}}{x} & ; x \neq 0 \\ \frac{1}{2} & ; x = 0 \end{cases}$$

x) If $x = 25 - 3p - p^2$ is the demand function, find the price elasticity of the demand when $p = 3$.

y) A manufacturer produces two products A and B. He has his machines in operation for 24 hours a day. Production of each unit of A requires 2 hours of processing time in M1 and 6 hours in machine M2. Production of each unit of B requires 6 hours of processing in machine M1 and 2 hours in machine M2. The manufacturer earns a profit of Rs. 50 on each unit of A and Rs.20 on each unit of B. How many units of each product should he produce in order to achieve maximum profit?

z) For the cost function $C(x, y) = 3x^2 + 2xy + y^2 + 10$, for two commodities x and y, find the marginal cost at x=1 and y=5.

Q.3 Attempt the following: **(5 x 4=20)**

a) In how many years the amount of money will be double the principal at simple interest of 12% p.a.

b) Find the equation of the line passing through (3,-2) and perpendicular to the line $x - 3y + 7 = 0$.

c) If the marginal revenue for a certain commodity is $MR = 4x^3 + 6x^2 + 10x + 1$ where x is the output, find the revenue function and average revenue function when $x = 10$.

d) Find $\frac{dy}{dx}$ if $i) y = x^3 + e^x - 5(3^x) + 15$

$ii) y = (x + 2)^2(x - 3)$

OR

Q.III Attempt the following: **(5 x 4=20)**

w) Find the amount received when a sum of Rs. 12000 is invested at 15% p.a. for 2 years if the interest is compounded quarterly.

x) If P(k,0) lies on the line through A(-2,1) and B(6,-2), find the ratio in which P divides segment AB and the value of k.

y) The demand function for a certain commodity is $p = 80 - 3x^2$. Find the consumers surplus when $x = 5$.

z) Find the derivative upto 3rd order of the function $f(x) = 10x^5 + 8x^3 - 5x^2 + 8x$.

Q.4 Attempt the following: **(5 x 4=20)**

a) Find the amount for the ordinary annuity with periodic payment as Rs.1000 at the rate of interest 10% p.a for 1 year if period of payment is quarterly.

b) Find the value of k for which the following set of lines are concurrent.
$x + y - 5 = 0, 4x - 3y = k, 7x - 8y + 10 = 0$.

c) Find the values of x for which the function $f(x) = 1 + 60x - 9x^2 - 2x^3$ is (i) increasing and (ii) decreasing.

d) Evaluate $i) \int \frac{x5^x + 3x - x10x}{x} dx$.

$ii) \int (5x + 2)^6 \, dx$

OR

Q.IV Attempt the following: **(5 x 4=20)**

w) If I wish to avail of Rs. 34,725 at the end of 3yrs. How much installments should I pay at the end of each year, given that the rate of interest is 15% to be compounded yearly.

x) Find the equation of the line joining the origin and the point of intersection of the lines $x + y = 5, x - y = 1$.

y) If $C = \frac{x^3 - 10x^2 - 300x}{3}$ is the cost function of a firm; x being the output, find the output at which marginal cost is minimum.

z) Evaluate $i) \int [(2x + 1)^5 - (3 - 5x)^2 + e^{7x} - 3^{6x}) \, dx$

$ii) \int (x^5 - 3(5)^x + \frac{5}{x}) dx$