

Vidya Vikas Mandal's
 Shree Damodar College of Commerce and Economics, Margao, Goa
 FYBCOM, Semester II, Semester End Examination-April 2015
MATHEMATICAL TECHNIQUES

Duration: 2 hours

Max. Marks: 80

1. All questions are compulsory.
2. Figures to the right indicate full marks
3. Use of calculators is allowed.

Q.1 Attempt the following:

(5 X 4 = 20)

- a. Find the equation of the line passing through (-3,5) and perpendicular to the line through the points (2,5) and (-3,6).
- b. Find the derivative of the following w.r.t x if
 - i) $y = \frac{x+1}{x}$
 - ii) $y = (\sqrt{x} + \frac{1}{\sqrt{x}})^2$
- c. If $f(x) = 3x + k$ and $f(1) = 7$, find k and $f(4)$.
- d. Evaluate the following integrals:
 - i) $\int \sqrt{4x+11} dx$
 - ii) $\int \frac{2x^2+x-3}{x} dx$

Or

Q.1 Attempt the following:

(5 X 4 = 20)

- w. The point P (7, a) lies on the line joining A(-5,2) and B(3,6). Find the ratio in which P divides AB and hence find a.
- x. Find the derivative of the following w.r.t x if
 - i) $y = (x^2 + 2)(x - 5)$
 - ii) $y = x^3 e^x - x^{-6}$
- y. If $f(x) = 2x + 7$ and $g(x) = x^2 + 1$, find $f(g(x))$ and $g(f(x))$.
- z. Evaluate the following integrals:
 - i) $\int_2^3 (e^x - 2x) dx$
 - ii) $\int_0^6 (x+1)(x-2) dx$

Q.2 Attempt the following:

(5 X 4 = 20)

- a. Find $\lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{x^2-4}$
- b. Find the values of the demand, for which the supply function $f(x) = x^3 - 6x^2 + 9x + 1$ is (i) increasing (ii) decreasing
- c. A toy manufacturer produces toy cars and toy planes, each of which must be processed through two machines A and B. Machine A has a maximum of 120 hours available and machine B has a maximum of 180 hours available. Manufacturing a car requires 6 hours in machine A and 3 hours in machine B. Manufacturing a plane requires 4 hours in machine A and 10 hours in machine B. If profits are Rs. 45 for a car and Rs. 55 for a plane. Formulate the L.P.P to maximize the profit. (no graph to be drawn)
- d. For the function $f(x, y) = x^2 + 2xy - y^2 + 10$. Find f_x and f_y at (1, 2).

OR

Q.11 Attempt the following:

(5 X 4 = 20)

- w. Examine the continuity of f at $x = 3$ if

$$f(x) = \begin{cases} x^2 + 1 & 0 \leq x \leq 3 \\ 3x + 1 & 3 < x \leq 5 \end{cases}$$
- x. The cost of manufacturing x toys is given by $C = x^2 + 5x + 5$, where x denotes the number of toys manufactured. Find the total cost, average cost and marginal cost when 10 toys are made.
- y. Solve graphically the following L.P.P
 Maximise $z = 9x + 13y$
 subject to

$$\begin{aligned} 2x + 3y &\leq 18 \\ 2x + y &\leq 10, \\ x &\geq 0, y \geq 0. \end{aligned}$$

z. The demand function for a certain commodity is given by

$$D(p_1, p_2) = 5 + 2p_1 p_2 - p_1^3.$$

Find the Marginal demand at $p_1 = 2$ and $p_2 = 6$.

Q.3 Attempt the following:

(5 X 4 = 20)

- Shirish invested Rs. 5000 in a bank as a short term deposit for 9 months at 5% to be compounded quarterly. Find the amount received by him on maturity.
- Show that (3,-5), (4, 3) and (11,-4) are the vertices of an isosceles triangle.
- If the Marginal Revenue function for a certain product is $MR = 4x^3 + 6x^2 + 10x + 1$ Find the Revenue function and Average Revenue when $x = 10$.
- A firm produces an output of x tons at a total cost $C = x^3 - 4x^2 + 7x$. Find the output at which the average cost is minimum.

OR

Q.III Attempt the following:

(5 X 4 = 20)

- In how many years, the amount of money will be double the principal at simple interest of 12% per annum?
- If $A(5,y)$ and $B(2,-3)$, find the possible values of y so that $I(A,B)=5$.
- The demand function for a certain commodity is $p = 100 - 5x$. Find the consumer's surplus at $x = 4$.
- If $y = e^x + \frac{1}{x} - 3x$. Find $\frac{d^2y}{dx^2}$.

Q.4 Attempt the following:

(5 X 4 = 20)

- Find the amount of an annuity of Rs. 6000, payable at the end of each quarter for 2 years, the interest rate being 8%, compounded quarterly.
- Find the equation of a line whose x-intercept is 4 and which is perpendicular to the line $x-2y+4=0$.
- If the demand function is given by $D = 25 - 3p - p^2$, Find price elasticity of demand when $p=2$.
- Evaluate: $\int \left[\frac{(3x-4) + \sqrt{(3x-4)+1}}{\sqrt{3x-4}} \right] dx$

Or

Q.IV Attempt the following:

(5 X 4 = 20)

- Find the present value of an ordinary annuity of Rs. 3500 per year for 3 years at 12% per annum.
- Find the equation of a line which passes through the point of intersection of the lines $x+2y-3=0$ and $3x+4y-5=0$ and which is perpendicular to the line $x-3y+5=0$.
- Show that $f(x) = x^5 - 5x^4 + 5x^3 - 1$ has maximum when $x=1$, a minimum when $x=3$.
- Evaluate: $\int_0^2 (3x^2 - x^3) dx$