

MATHEMATICAL TECHNIQUES

Duration: 2 hours

Max. Marks: 80

Instructions: 1) All questions are compulsory (choice is internal)

2) Start each new question on a fresh page

3) Figures to the right indicate full marks

4) Nonprogrammable calculators are allowed.

Q.1 Attempt the following:

(5 x 4 = 20)

a) Prove that the following statements are equivalent:

$$\sim(p \wedge q \wedge r) \equiv (\sim p \vee \sim q \vee \sim r)$$

b) Let $A = \{x | x^2 - 7x + 12 = 0\}$, $B = \{x | x^2 - 5x + 6 = 0\}$ and $C = \{x | x^2 - 3x + 2 = 0\}$.Verify $A \cap (B - C) = (A \cap B) - (A \cap C)$

c) Solve the following using Cramer's rule

$$5a - 2b + 3c = 9, \quad 3a + 4b + 2c = 10, \quad a + b + c = 2$$

d) Find the three numbers in A.P. such that their sum is 27 and the third number is double the first number.

OR

Q.1 Attempt the following:

(5 x 4 = 20)

w) Prove that $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$ is a tautologyx) Let $U = \{x | x \in \mathbb{N}, x \leq 10\}$ be the universal set and $A = \{2, 3, 5, 7\}$ and $B = \{1, 3, 5, 7, 9\}$. State and verify De Morgan's Law

y) Solve the following determinant equation

$$\begin{vmatrix} 5 & -3 & 7 \\ 2 & 1 & 2 \\ 9 & -1 & x \end{vmatrix} = 0$$

z) For a G.P. $T_3 = 20$ and $T_7 = 320$, find first term and T_{10} .

Q.2 Attempt the following:

(5 x 4 = 20)

a) A class has 6 girls and 5 boys. If 4 persons out of these are to be selected, find the total no. of choices if:

i. There is no restriction on gender

ii. 3 boys and 1 girl is to be selected

b) Find the value of x and y from the matrix equation

$$\left\{ 4 \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \end{bmatrix} - 2 \begin{bmatrix} 1 & 3 & 1 \\ 2 & 3 & 8 \end{bmatrix} \right\} \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

c) What number must be added to each term of the ratio 5:37 to make it equal to 1:3?

d) Find cofactors of the elements of the following matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 3 \\ 5 & 7 & 4 \end{bmatrix}$$

OR

Q.11 Attempt the following:

(5 x 4 = 20)

w) How many arrangements can be made with the letters of the word "MATHEMATICS" and in how many of them vowels occur together?

x) Find the inverse of

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

y) The ratio of two numbers is 5:6. On adding 3 to each of these numbers, the ratio becomes 6:7. Find the numbers.

z) Show that the equations $2x + 3y = -1$, $x + 2y = 1$, $x + y = -2$ are inconsistent.

Q.3 Attempt the following:

(5 x 4 = 20)

a) In a recent survey of 400 students in a college, 100 were listed as smokers and 150 were listed as chewers of gum. 75 were listed as both smokers and chewers of gum.

- c) If $A = \begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ find the matrix X such that $5A + 3B + 2X = 0$
- d) If the sum of three nos. in an A.P. is 18 and the sum of their squares is 116 find them.

OR

Q.III Attempt the following:

- w) In a survey of 30 students, it was found that 19 take mathematics, 17 take information technology and 11 take costing, 7 take mathematics and costing, 12 take mathematics and information technology, 5 take information technology and costing and 2 take all three course. Find the no. of students who took
- Mathematics but not costing.
 - Exactly 2 of the three course.

x) ${}^nC_8 = {}^nC_6$ find nC_2

y) If $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ and $A + 2B = A^2$ find B

- z) If the fifth term of a G.P. is 81 whereas its second term is 24. Find the n^{th} term of the series and sum of its first four terms.

Q.4 Attempt the following:

- a) Akhil saves 20 % of his earnings. Due to increase in cost of living by 15% he could now save only Rs.780. Find the monthly earnings.
- b) Suppose the statements p, q, r, s are assigned truth values F, T, T, F respectively, find the truth values of each of the following:

1) $(p \wedge q) \rightarrow [(\sim r) \vee s]$, 2) $(p \rightarrow \sim s) \vee (r \wedge q)$, 3) $r \rightarrow s \vee (p \rightarrow q)$, 4) $q \rightarrow (\rightarrow s)$, 5) $\sim p \wedge \sim s \vee q$

c) If $A = \begin{bmatrix} 3 & 3 & 4 \\ 2 & 3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ find the adjoint of the matrix A .

- d) Out of 5 males and 6 females, a committee of 5 is to be formed. Find the no. of ways in which it can be done so that among the person chosen in the committee there are:
- 3 males and 2 females
 - 2 males

OR

Q.IV Attempt the following:

- w) The list price of an article is Rs.120. If a discount of 6% is allowed. Find the amount payable by the customer.
- x) Test the validity of the following argument:
"Either the country should import more goods or it should start manufacturing goods. The country is not at all thinking of imports. Therefore the country is going to start manufacturing goods."

y) If $A = \begin{bmatrix} 4 & 0 & 3 \\ 1 & 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2 \\ 1 & 3 \\ 2 & 1 \end{bmatrix}$ show that AB is non-singular.

- z) How many different arrangements can be made by using all the letters of the word
- MONDAY
 - ORIENTAL?

How many of these arrangements begin with A and end with N?