Duration: 2 hours   **Total Marks: 50**

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

1. **Answer the following**
   a. **Convert**   **Marks(1 X 5)**
      i. $(1011101)_2$ to decimalnumber
      ii. $(7152)_{10}$ to octal number
      iii. $(2407)_8$ to decimal number
      iv. $(5381)_{10}$ to binary number
      v. $(BD2C)_{16}$ to decimal number

   b. **Answer the following questions.**   **Marks(1 X 5)**
      i.   Write the converse and inverse of $(p \lor q) \rightarrow \sim p$
      ii. Give an example of a relation that is symmetric.
      iii. If A = {1,2}, B={2,3}, C={3,5},find $A X (B \cap C)$
      iv. In how many ways can a student choose 5 courses out of 9 courses if 2 courses are compulsory?
      v.   Simplify $\bar{x} + (x + \bar{y} + \bar{z})$ using Boolean algebra

2. **Answer the following (Any two)**   **Marks(5 X 2)**
   a. Prove by the principle of mathematical induction
      $4+8+12+\ldots\ldots+4n=2n(n+1)$
   b. Prove that $p \oplus q \equiv (p \land \sim q) \lor (\sim p \land q)$
   c. Simplify the boolean expression $a + \bar{a} .(a +b) + b.c$

3. **Answer the following (Any two)**   **Marks(5 X 2)**
   a. In a group of 70 persons, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. Draw an appropriate venn diagram.
      (1) How many people like both coffee and tea?
      (2)How many people like coffee but not tea?

   b. If A={1,2,3,4}, B={2,4,6,9} , C={3,4,5,6}.
      Find (i)$A \cup B \cap C$) (ii)$A \cap (b \cup C)$ (iii)$(B-C) \cap A$ (iv)$A \triangle B$(v)P(A)

   c. Prove that the relation R on the set of integers $\mathbb{Z}$ defined as R={ (x,y)/x-y is divisible by 3, $x \in \mathbb{Z}, y \in \mathbb{Z}$} *is* an equivalence relation.

4. **Answer the following (Any two)**   **Marks(5 X 2)**

   a. Find the coefficient of x^4 in $(1-2x^2)^4$.

   b. 7 persons including 2 ladies, 3 gents and 2 children go for a movie. How many different seating arrangements can be done in a line so that i) anybody can sit anywhere
   (ii) the ladies occupy the end seats
   (iii) gents occupy the end seats as well as the middle seat.

   c. An organization consists of 9 members of which 4 are doctors. A selection of 4 persons is to be done amongst these members. Find how many selections will have
   (i) no doctor
   (ii) Exactly 2 doctors
   (iii) At most one doctor

5. **Answer the following (Any two)**   **Marks(5 X 2)**

   a. If $f(x) = x^2 + 1$; $0 \leq x \leq 2$
   $\qquad\quad = 3x - 5$; $2 \leq x \leq 4$
   $\qquad\quad = 4x + 3$; $4 \leq x \leq 8$
   Find $f(1), f(2), f(3), f(4.1)$ and $f(6)$.

   b. Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^3 + 1$ is one-one and onto.

   c. Let $L_1 = \{ ab^3, ba^3 \}$ and $L_2 = \{ a^3, b^3, ab \}$ be 2 languages over an alphabet $\sigma = \{a, b\}$. Find $L_1 L_2$, $L_2 L_1$, L_1^2, L_2^2.

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