DISCRETE MATHEMATICS

Duration: 2 Hours   **Total Marks: 50**

Instructions:
1. *Figures to the right indicate maximum marks*
2. *All questions are compulsory*

Q .1 Answer the following

     a) Answer the following questions   **(1X5=5)**

         i.   Write the truth value for $(p \uparrow q) \vee (\sim q)$, if $p = F, and\ q = T$

         ii.   Find the Range of f where $f : \mathbf{R} \longrightarrow \mathbf{R}$ defined by

$$f(x) = \begin{cases} 1 & if\ x > 0 \\ 0 & if\ x = 0 \\ -1 & if\ x < 0 \end{cases}$$

         iii.   Draw XOR gate as the combination of OR, AND and NOT gate

         iv.   If A= $\{x\epsilon N|\ 1 < x < 9\}$ and B= $\{x\epsilon N|\ x\ is\ even\ and\ x < 10\}$

            Write A−B and B−A

         v.   Find the relation between $^{n}C_{r}$ and $^{n}P_{r}$

     b) Solve the following   **(1X5=5)**

         i.   Convert $(1010111)_2$ into Decimal number

         ii.   Convert 77.125 into Binary number

         iii.   Convert $(56)_8$ into Decimal number

         iv.   Convert $(AFD5)_{16}$ into Decimal number

         v.   Convert 0.125 in to Binary number

Q .2 Answer the following (Any two)   **(5X2=10)**

     a) Use principle of mathematical induction to prove that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}, \quad n\epsilon N$$

     b) Write the binomial expansion of $(2x + y)^{10}$ and find the coefficient of $x^7 y^3$

     c) Prepare truth table for $p \downarrow (\sim q)$

Q .3 Answer the following (Any two)   **(5X2=10)**

     a) Prove the following using properties of Boolean algebra

         i. $\overline{(x + y)} + \overline{(x + \bar{y})} = \bar{x}$

         ii. $(a + b) \cdot (\bar{a} + \bar{b}) = \bar{a} \cdot b + a \cdot \bar{b}$

b) Show that $(p \rightarrow q)$ is equivalent to $(\sim p \vee q)$.

c) A survey was conducted in a city to study the preference of three brands of products, A, B and C. The total number of persons interviewed were 200. 92 people said that they use A, 86 use B and 83 use C. also 25 of them use A and B, 27 use A and C, 26 use B and C and 9 use all the three. Find the number of people using

    i.   A only

    ii.   B only

    iii. A or B

    iv. A and B

    v.   People using exactly two brands

(5X2=10)

Q .4 Answer the following (Any two)

a) If $U = \{1,2,3,4,5,6,7,8,9,10\}$, U is a universal set and A= $\{2,3,5,7\}$ and B = $\{2,4,6,8,10\}$, then find

    i. $\acute{A}$

    ii. $\acute{A} \cap \acute{B}$

    iii. $A - B$

    iv. $B - A$

    v. $\acute{A} \cup \acute{B}$ (given $\acute{A}$ means complement of A)

b) Prove that $^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

c) Prove that the relation R defined on **R**(the set of all real numbers) as $a\,R\,b \Leftrightarrow a \leq b$ is a Partial order relation

(5X2=10)

Q .5 Answer the following (Any two)

a) Define composition of two functions. Consider the functions $f, g : \mathbf{R} \longrightarrow \mathbf{R}$ defined by $f(x) = 2x^2 + 5$ and $g(x) = x^3$ find fog, gof, fof, gog

b) Let A=$\{1,2,3,4,5,6,7,8\}$ and a relation on A be R= $\{(x,y) \epsilon A \times A \mid y = x + 2\}$

    i.   Write R in Roster form

    ii. Prove that R is not symmetric

    iii. Prove that R is not transitive

    iv. Prove that R is not reflexive

    v.   Write A×A

c) Find the languages L(G) generated by the grammar with V=$\{S, A, B\}$, $\sigma = \{x, y\}$ and

$$P = \left\{ S \rightarrow xB, B \rightarrow y, B \rightarrow yA, A \rightarrow xB \right\}$$

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