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Shree Damodar College of Commerce & Economics Margao Goa
F.Y.BCA, Sem II, End Semester Examination, April 2017

DISCRETE MATHEMATICS

Duration: 2 Hours

Total Marks: 50

Instructions:

1. Figures to the right indicate maximum marks
2. All questions are compulsory

Q.1 Answer the following

a) Answer the following questions (1X5=5)

- i. If p, q, r and s are assigned the truth values T, F, T and F respectively, then write the truth value of $(p \wedge q) \leftrightarrow (\sim r \vee \sim s)$
- ii. If $f(x) = x^2$ and $g(x) = 2x^2 + 4$ then find $g \circ f$, where $f, g: \mathbb{R} \longrightarrow \mathbb{R}$
- iii. Draw NOR gate as the combination of OR and NOT gate
- iv. If $A = \{x \in \mathbb{N} \mid (x - 2)(x - 5)(x - 1) = 0\}$ and $B = \{x \in \mathbb{N} \mid x^2 - 7x + 10 = 0\}$
Write $A \cup B$ and $A \cap B$
- v. In how many ways can you arrange six students in a row so that three particular students are always together

b) Solve the following (1X5=5)

- i. $(1100011)_2 + (100111)_2$
- ii. $(111101)_2 - (10111)_2$
- iii. Convert $(10000101)_2$ into Decimal number system
- iv. Convert 648 into octal form
- v. Convert $(AED5)_{16}$ into Decimal number system

Q.2 Answer the following (Any two) (5X2=10)

- a) Use principle of mathematical induction to prove that

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}, \quad n \in \mathbb{N}$$

- b) Write the binomial expansion of $(2x + 1)^{10}$ and find its 7th term and 9th term in the Expansion
- c) Prove that $(p \uparrow q) \oplus (p \uparrow q)$ is a contradiction

Q.3 Answer the following (Any two) (5X2=10)

- a) Prove the following using properties of Boolean algebra

- i. $x \cdot x = x$
- ii. $x + x = x$

- b) Construct the truth table for $\sim(p \wedge q) \leftrightarrow (\sim p \vee \sim q)$
- c) A town has a total population of 40000 out of which 400 people own cars 10,000 people own bicycles and 300 own both cars and bicycles. How many people in the town do not own either? How many own only bicycles? And how many own only cars?

Q .4 Answer the following (Any two)

(5X2=10)

- a) if U is a universal set and A and B are any two subsets of U , then define the following $A \cup B, A \cap B, A - B, \bar{A}, A \times B$ where \bar{A} is the complement of A . (Write in set builder form)
- b) in how many ways can the letters of the word HEXAGON be arranged so that
- H appears in the beginning
 - H and G appear at the ends
 - XAG are always together
 - XAG are never together
 - H never appears in the beginning
- c) Prove that the relation R defined on \mathbf{R} (the set of all real numbers) as $a R b \Leftrightarrow a = b$ is an Equivalence relation

Q .5 Answer the following (Any two)

(5X2=10)

- a) Consider a function $f: \mathbf{R} \longrightarrow \mathbf{R}$ defined by $f(x) = 2x + 3$
- Prove that f is one-one
 - Prove that f is onto
 - Prove that f is a bijection
 - Find inverse function of f
- b) Let $A = \{1, 2, 3\}$ and a relation on A be
- $$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (1, 3)\}$$
- prove that
- R is not reflexive
 - R is symmetric
 - R is not transitive
 - Write the missing element that makes R an Equivalence relation
 - Write $A \times A$
- c) Find the languages $L(G)$ generated by the grammar with $V = \{S, A, B\}$, $\sigma = \{x, y, z\}$ and

$$P = \{S \rightarrow xSy, xS \rightarrow Ax, Axy \rightarrow z\}$$

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